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YBMA News

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The Newsletter of the Yorkshire Branch of the Mathematical Association

Many readers will recall former prime minister Rishi Sunak's '*Maths until 18*' announcement in early 2023. Children leaving school and going out into a world full of data and statistics were to be equipped with the skills they would need. A worthy ambition was the consensus of opinion in the maths education community, but how would it be achieved? There was no clarity on the courses students would follow. About half of 16 to 19-year-olds already study maths in one form or another. The plan would not make A-levels compulsory and no new qualifications were in the pipeline. How would it affect students studying the humanities or the creative arts? Would students continue to resit GCSE maths until they achieved grade 4 or above?

In 2024, 71.2% of students aged 16 gained a GCSE pass in mathematics. Just 18.6% of entrants aged 17+ achieved this and presumably nearly all were re-taking the exam. The current system is not working. It has been suggested that a driving test or passport-style certification might instil students with greater confidence. Perhaps a step in the right direction, but by itself not enough.

Maths education is failing UK students, so writes Bobby Seagull in the Financial Times*. Yet there are those who maintain that it's all a fuss about nothing, little of what is taught in maths lessons is relevant to daily life and that they themselves don't feel the need to improve their grasp of the subject.

The National Centre for Excellence in the Teaching of Mathematics (NCETM) was set up in 2006 to support mathematics teachers and improve the teaching of mathematics. Charlie Stripp has been its Director since 2013. He has kindly agreed to be the speaker at the next YBMA meeting. I am looking forward to hearing what he has to say.

** Access to the article does not require a subscription.*

YBMA Officers 2024-25

President: Lindsey Sharp (lindseyelizab50@hotmail.com)

Secretary & Newsletter: Bill Bardelang (rgb43@gmx.com)

Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

Our next Meeting

Wednesday, 9 October 2024
7pm for 7:30pm

MALL 1, School of Mathematics
University of Leeds

Charlie Stripp MBE
Chief Executive, MEI
NCETM National Director
President of the Mathematical Association

Key challenges in Maths Education and how we might address them (and some nice bits of maths!)

Maths education has never been more important to individuals or to society. There are some successes to celebrate, but there are also some key challenges that are limiting our ability to improve maths education in the UK, in particular:

- *Public attitudes to maths and maths education*
- *A chronic shortage of secondary maths teachers*
- *A poor track record of using technology to enhance teaching and learning in mathematics*

Charlie will discuss these challenges and what might be done to address them. He will also share some nice maths problems!

Christmas Quiz

Wednesday, 4 December 2024
7pm for 7.30pm

MALL 1, School of Mathematics
University of Leeds

Seasonal Food and Drink! Prizes!

Last year's format was a great success, so again we hope many of you will bring along a round of questions, presented in any way you choose. Suggested time allowance: 5-10 minutes.

A Date for your Diary

Wednesday 2nd April 2025, 2:30pm
at the University of Leeds

W P Milne Lecture for Sixth Formers

Katie Steckles

The Mathematics of Paper

The W. P. Milne lecture forms part of the Key Stage 5 Maths Day held annually at the University of Leeds.

More details at

www.stem.leeds.ac.uk/ks5mathsday

Mathematics in the Classroom

Number Sequences and Recurring Decimals

When we write a number $0<N<1$ as a decimal $0.a_1a_2a_3\ldots$ we are expressing it as a power series $\sum_i a_i t^i$, where $t=\frac{1}{10}$. The coefficients a_i can be thought of as an integer sequence associated with N. For rational numbers N the associated sequence may be finite or infinite. In the infinite case the sequence can be described by a simple recurrence relation. For example

$\frac{91}{370}=0.2\dot{4}5\dot{9}$ and the associated infinite sequence has terms $a_1=2, a_2=4, a_3=5, a_4=9$, and $a_n=a_{n-3}$ for $n\geq 5$.

So far we have implicitly assumed that all terms satisfy $0\leqslant a_i\leqslant 9$. Relaxing this restriction to $0\leqslant a_i$ allows us to consider a variety of well-known sequences and ask whether they are associated with a rational number. We will take the sequence of odd numbers 1, 3, 5, 7, 9, 11, 13, … as an example. Its associated decimal would then be N = 0.(1)(3)(5)(7)(9)(11)(13)… .

We can rewrite N in its 'standard' decimal form by repeatedly subtracting 10 from each a_i while adding 1 to the corresponding a_{i-1} until $a_i\leqslant 9$. Thus

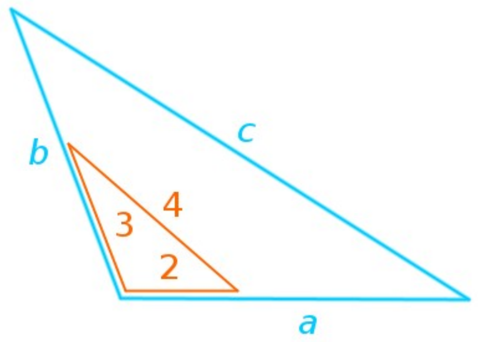
N = 0.(1)(3)(5)(7)(9)(11)(13)… = 0.(1)(3)(5)(8)(0)(2)(4)… = 0.1358024… .

This can be shown to be the decimal equivalent of the rational number $\frac{11}{81}$ and has a recurring sequence of 9 digits.

Show that the following sequences are associated with a rational number. Find this number and the length of the recurring sequence of digits in its standard decimal form.

a) 1, 4, 9, 16, 25, … the sequence of square numbers,
b) 1, 1, 2, 3, 5, 8, 13, … the Fibonacci sequence.

Integer Triangle Families - Solution



In the May Newsletter we asked readers to find integer-sided triangles with the same obtuse angle as the 2, 3, 4 triangle. Denoting this obtuse angle by θ and making use of the cosine rule:

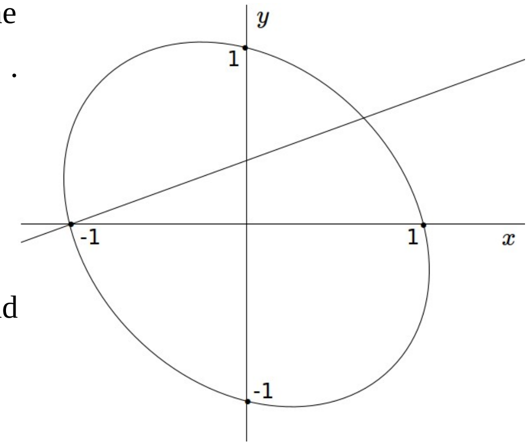
$$\cos(\theta)=\frac{a^2+b^2-c^2}{2ab}=\frac{2^2+3^2-4^2}{2\times 2\times 3}=-\frac{1}{4}$$

and thus $a^2+b^2+\frac{1}{2}ab=c^2$.

In the 2023 W.P. Milne lecture we were shown a method for finding integer solutions of $a^2+b^2=c^2$. We will use the same approach here. Dividing our equation throughout by c^2 we obtain

$$x^2+y^2+\frac{1}{2}xy=1 \quad , \quad \text{where} \quad x=\frac{a}{c} \quad , \quad y=\frac{b}{c} \quad .$$

The graph of the resulting equation is an ellipse, as shown in the diagram below. Our aim is to find the rational points of the ellipse in the first quadrant.



Consider the points of intersection of the ellipse with the line $y=\frac{l}{m}(x+1)$ where l,m are co-prime integers, $0<l<m$. Eliminating y between the simultaneous equations gives

$$x^2+\frac{l^2}{m^2}(x+1)^2+\frac{1}{2}\frac{l}{m}x(x+1)=1$$

This equation factorises, one factor leading to $(-1,0)$ and the other to

$$x=\frac{m^2-l^2}{m^2+l^2+\frac{1}{2}ml} \quad , \quad y=\frac{2ml+\frac{1}{2}l^2}{m^2+l^2+\frac{1}{2}ml} \quad .$$

Choosing $a=2(m^2-l^2)$, $b=4ml+l^2$, $c=2(m^2+l^2)+ml$ ensures we obtain integer values for a,b,c and dividing by their highest common factor gives the corresponding primitive solution.

Because the relation between a and b is symmetrical, there are two possible pairs l,m for each primitive solution. For example, both $l=4,m=5$ and $l=1,m=11$ lead to the triangle with sides 3, 16, 17. In the Pythagorean case we usually insist that l,m are of opposite parity to avoid this situation, but there seems be no such simple remedy here.